Introduction

A **mirror image** is a reflected duplication of an object that appears identical but reversed. As an optical effect it results from reflection off of substances such as a mirror or water. The reflection transformation is called derived transformation because it is derived from general transformation.

Shear mappings carry areas into equal areas and volumes into equal volumes, as they preserve the width, length, and etc. of parallelograms.

Reflection Transformation

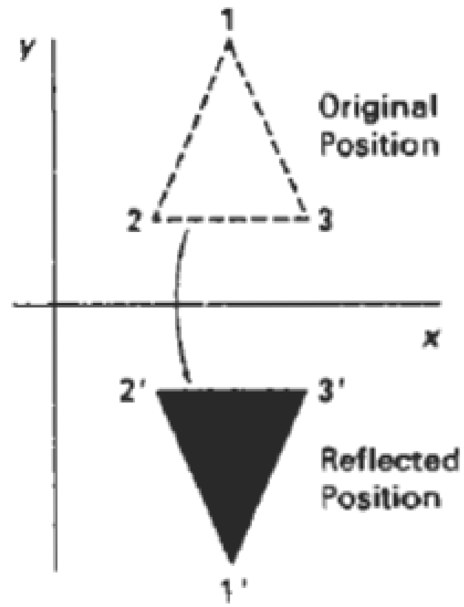


Figure 5-16
Reflection of an object about the x axis.

Reflection about the line $y = 0$, the x axis, is accomplished with the transformation matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5-48}$$

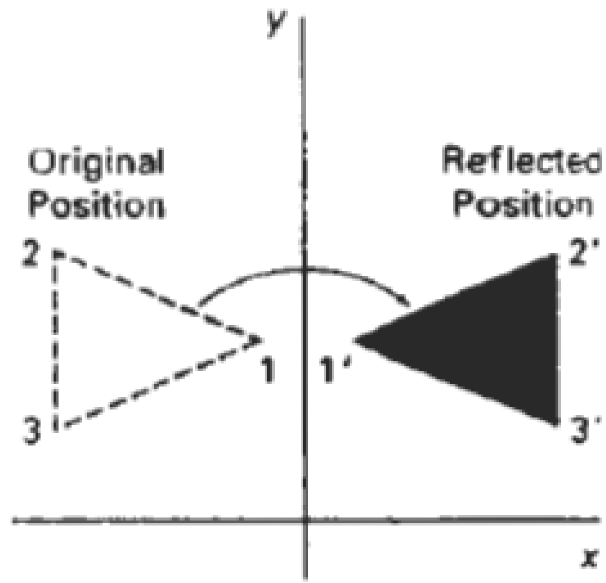


Figure 5-17
Reflection of an object about the y axis.

A reflection about the y axis flips x coordinates while keeping y coordinates the same. The matrix for this transformation is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5-49}$$

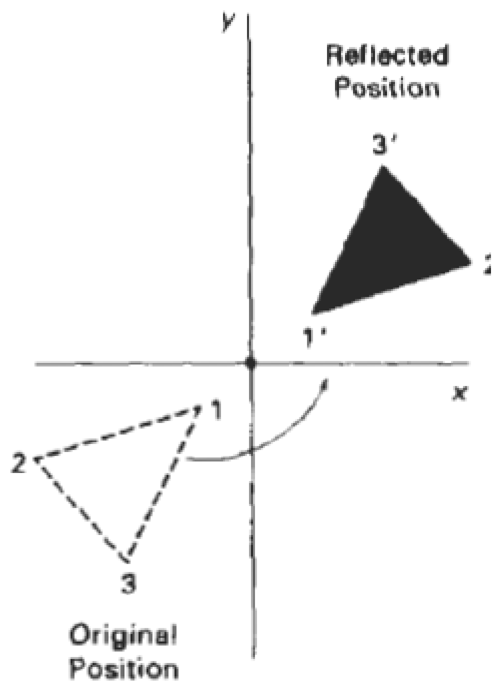
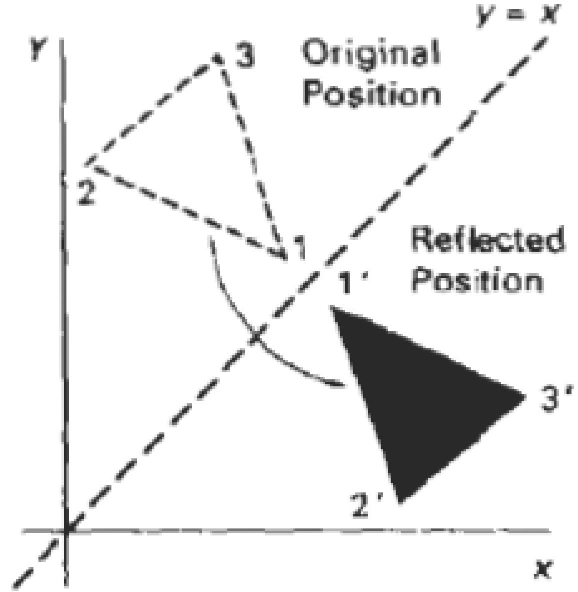


Figure 5-18
Reflection of an object relative to an axis perpendicular to the xy plane and passing through the coordinate origin.

We flip both the x and y coordinates of a point by reflecting relative to an axis that is perpendicular to the xy plane and that passes through the coordinate origin. This transformation, referred to as a reflection relative to the coordinate origin, has the matrix representation:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5-50}$$



If we chose the reflection axis as the diagonal line $y = x$ (Fig. 5-20), the reflection matrix is

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5-51}$$

Reflection about line Y=-X

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shear

A transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called a **shear**. Two common shearing transformations are those that shift coordinate x values and those that shift y values.

An x-direction shear relative to the x axis is produced with the transformation matrix

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5-53}$$

which transforms coordinate positions as

$$x' = x + sh_x \cdot y, \qquad y' = y \tag{5-54}$$

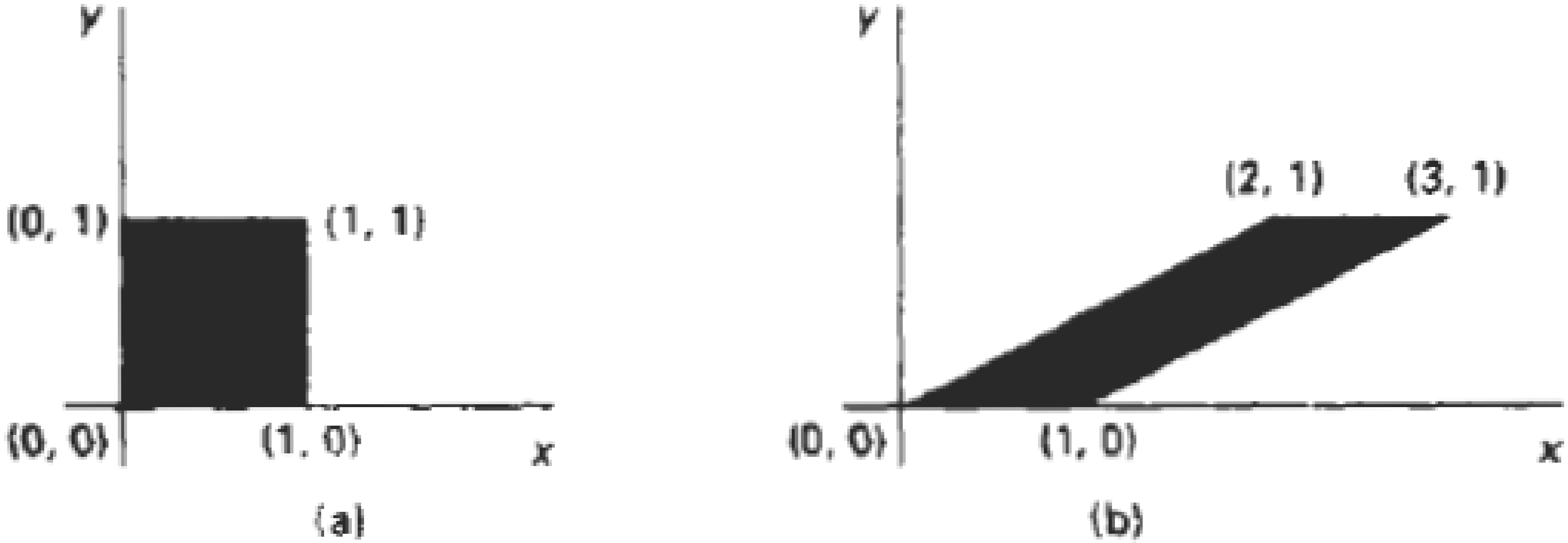


Figure 5-23
A unit square (a) is converted to a parallelogram (b) using the x-direction shear matrix 5-53 with $sh_x = 2$.

We can generate x-direction shears relative to other reference lines with

$$\begin{bmatrix} 1 & sh_x & -sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5-55}$$

with coordinate positions transformed as

$$x' = x + sh_x(y - y_{ref}), \qquad y' = y \tag{5-56}$$

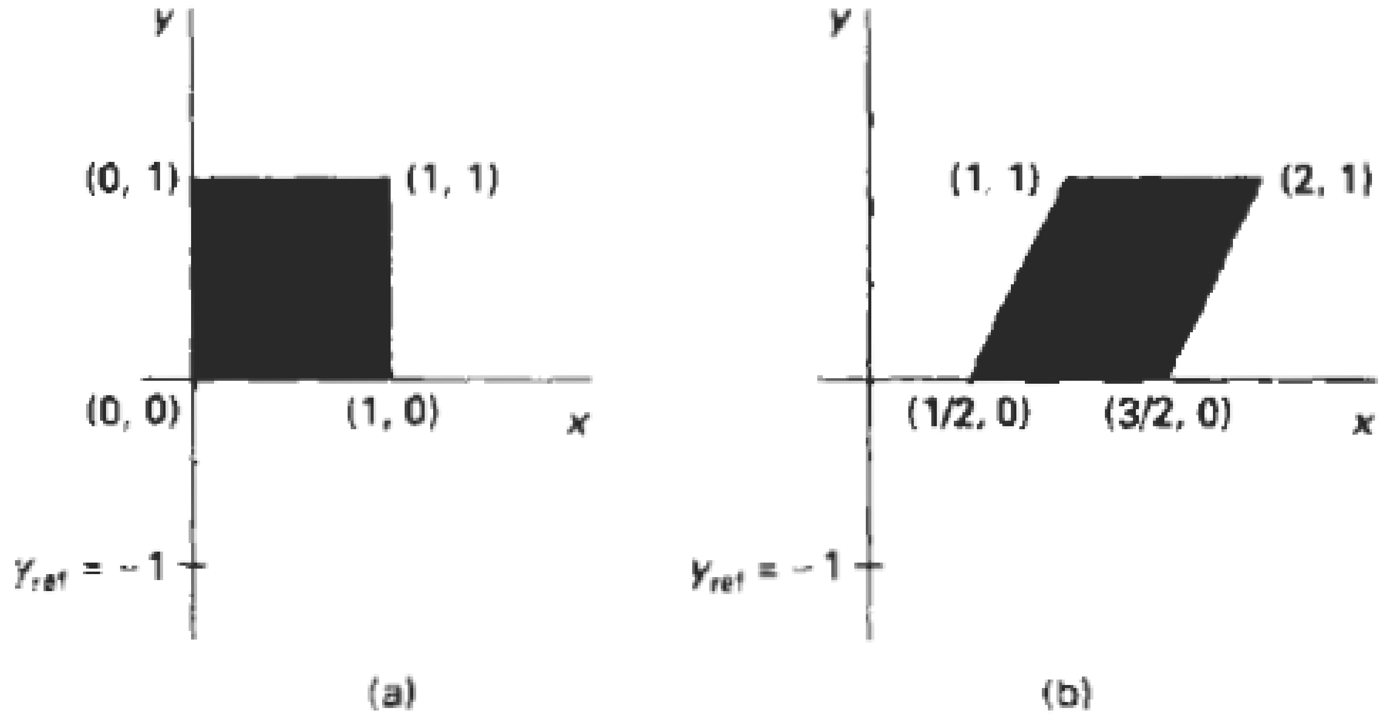


Figure 5-24
A unit square (a) is transformed to a shifted parallelogram (b) with $sh_x = 1/2$ and $y_{ref} = -1$ in the shear matrix 5-55.

A y-direction shear relative to the line $x = x_{ref}$ is generated with the transformation matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y \cdot x_{ref} \\ 0 & 0 & 1 \end{bmatrix} \tag{5-57}$$

which generates transformed coordinate positions

$$x' = x, \qquad y' = sh_y(x - x_{ref}) + y \tag{5-58}$$

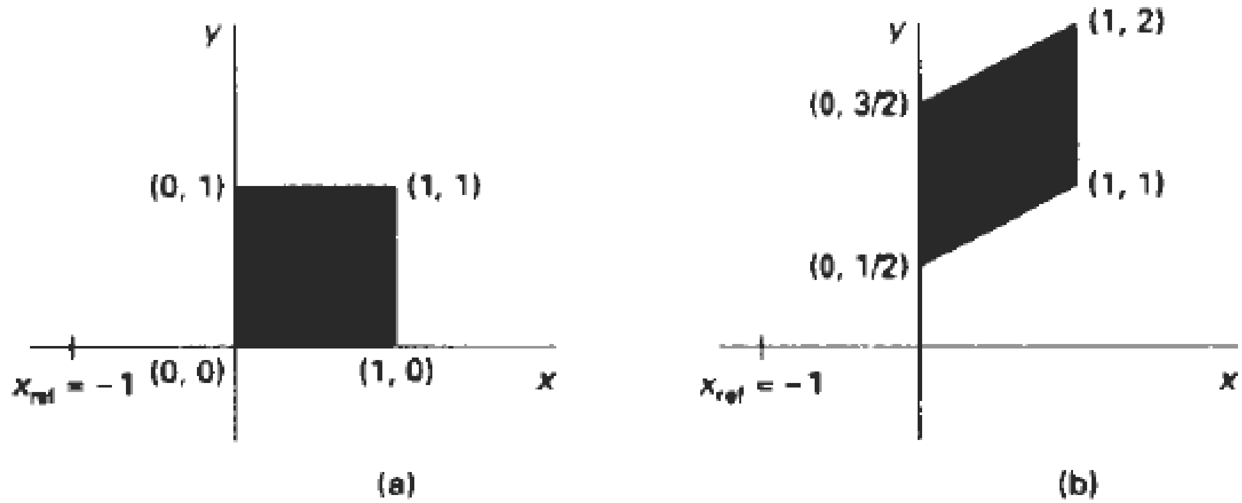


Figure 5-25
A unit square (a) is turned into a shifted parallelogram (b) with parameter values $sh_y = 1/2$ and $x_{ref} = -1$ in the y-direction using shearing transformation 5-57.

Application

Several techniques can be used to move signals in the time-frequency distribution. Similar to computer graphic techniques, signals can be subjected to horizontal shifting, vertical shifting, dilation (scaling), shearing, and rotation. These techniques can help to save the bandwidth with proper motions apply on the signals. Moreover, filters with proper motion transformation can save the hardware cost without additional filters.